A ROBUST AND EFFICIENT METHOD FOR THE FREQUENCY DOMAIN ANALYSIS OF NON-UNIFORM, LOSSY MULTI-LINE TRANSMISSION STRUCTURES

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ABSTRACT

A new efficient and robust method of moment-based formulation for the analysis of non-uniform, lossy multi-line transmission structures is proposed. The stability and speed of this method are demonstrated by analyzing different non-uniform structures. The performances obtained show the proposed approach to be suitable for CAD applications.

INTRODUCTION

In today's technological drive towards ever higher performing microwave circuits and systems, non-uniform transmission structures, both single and multiple coupled lines, are being used more and more. In recent years, several different approaches have been proposed for the analysis of these structures [1-4]. While single non-uniform lines have received most of the attention, resulting in several alternative modeling techniques, the same cannot be said of multiple coupled lines where significantly less work has been carried out and where no CAD-suitable methods are available. In fact, aside from [1], where several piece-wise uniform multi-conductor transmission line segments have been cascaded, or [2, 3], where a moment method approach with spatially-fixed basis functions has been suggested, little other literature is available on the subject. Moreover, both of these approaches have limitations that make them impractical CAD tools, particularly when analyzing a large number of lines and at high frequencies.

In this paper a new efficient and robust approach for the calculation of S-parameters of lossy, non-uniform multi-line structures is proposed. This approach is based on the method of moments in the frequency domain with a particular choice of the expanded variable and a special set of basis functions that depend on the modal propagation constants of the structure. These functions build-in the frequency dependence of the solution. A detailed description of the proposed approach is first given then several results are presented and compared to published results or to commercial circuit simulators.

FORMULATION

The distribution of the current and voltage waves in coupled line structures is given by the well-known telegrapher's equations:

$$\begin{cases} \dfrac{\partial V_j(f,z)}{\partial z} = -\sum_{k=1}^{N} Z^{jk}(f,z).I_k(f,z) \\ \dfrac{\partial I_j(f,z)}{\partial z} = -\sum_{k=1}^{N} Y^{jk}(f,z).V_k(f,z) \end{cases} \tag{1}$$

where a harmonic time dependence of frequency *f* has been assumed and where $Z^{jk}(f,z)$ and $Y^{jk}(f,z)$ are the entries of the frequency dependent per-unit length impedance and admittance matrices $Z(f,z)$ and $Y(f,z)$ at a position z along the lines. These matrices, which are given in terms of the line-parameter matrices $R(f,z)$, $L(f,z)$, $G(f,z)$, $C(f,z)$ by $Z(f,z) = R(f,z) + j2\pi fL(f,z)$ and $Y(f,z) = G(f,z) + j2\pi fC(f,z)$, are assumed to be known at some discrete positions along the propagation axis z.

In [2, 3] a moment-method based approach was proposed where *both* the current and voltage waves on each conductor were expanded in terms of Chebychev polynomial. In [4], another method of moments-based approach, which also expands *both* the current and voltage waves, was proposed but with new frequency varying basis functions made of forward and backward propagating waves:

$$\begin{cases} V(z) = \sum_{i=1}^{nb} a_i e^{-j\beta_i z} + b_i e^{j\beta_i z} = \sum_{i=1}^{nb} a_i F_i(z) + b_i B_i(z) \\ I(z) = \sum_{i=1}^{nb} c_i e^{-j\beta_i z} + d_i e^{j\beta_i z} = \sum_{i=1}^{nb} c_i F_i(z) + d_i B_i(z) \end{cases} \tag{2}$$

The fact that both V and I are independently expanded in terms of basis functions builds in redundancy in the system since both quantities are already coupled through the telegrapher's equations. The redundancy thus introduced will lead, as will be shown below, to ill-conditioned matrices.

In order to make full use of the coupling between V and I, a new method of moment approach is proposed. First, the two equations in (1) are combined into a single second order differential equation in one of the variables. For the voltage vector $\overline{V}$, this equation reads:

$$Z\frac{\partial^2\overline{V}}{\partial^2 z}-\frac{\partial Z}{\partial z}\frac{\partial\overline{V}}{\partial z}-Z^2Y\overline{V}=\overline{0}_N \qquad (3)$$

where Z, Y and V are frequency and z-dependent quantities.

Second, a method of moments approach is used to solve (3) where only $\overline{V}(f,z)$ is expended in terms of forward, F_n, and backward, B_n, propagating waves as follows:

$$\overline{V}(f,z)=\sum_{n=1}^{N_g}\overline{a}_n\,F_n(z,f)+\overline{b}_n\,B_n(z,f) \qquad (4)$$

where a_n^j and b_n^j are the unknown coefficients associated with the nth basis function representing the voltage on the jth line, and N_g is the number of forward and backward waves used. The frequency dependent basis functions, F_n and B_n, are given by $F_n(z,f)=e^{-\gamma_n(f)z}$ and $B_n(z,f)=e^{+\gamma_n(f)z}$. The set of $\gamma_n(f)$, $n=1,\ldots N_g$, corresponds to the propagation constants of all the modes supported by the N-line structure computed at different positions along the z axis. Since $\gamma_n(f)$ can in general be complex, $\gamma_n=\alpha_n+j\beta_n$, the loss in the coupled lines is automatically accounted for in the solution.

Next, substituting (4) into (3) we obtain:

$$\sum_{n=1}^{N_g}\left(Z\gamma_n^2+Z_p\gamma_n-Z^2Y\right)\overline{a}_n e^{-\gamma_n z}+\sum_{n=1}^{N_g}\left(Z\gamma_n^2-Z_p\gamma_n-Z^2Y\right)\overline{b}_n e^{\gamma_n z}=\overline{0}_{N_g} \qquad (5)$$

where $Z_p=\dfrac{\partial Z(f,z)}{\partial z}$.

Finally, testing equation (5) with F_m and B_m (m=1,2.N_g-1), leads to a system of $2N*(N_g-1)$ equations in $2N*N_g$ unknowns. The entries in this matrix equation are given by the following inner products:

$$\left\langle[Z]e^{\pm\gamma_n z},e^{\pm\gamma_m z}\right\rangle,\ \left\langle\left[Z_p\right]e^{\pm\gamma_n z},e^{\pm\gamma_m z}\right\rangle \text{ and } \left\langle[Z]^2[Y]e^{\pm\gamma_n z},e^{\pm\gamma_m z}\right\rangle \qquad (6)$$

where $\langle f,g\rangle=\int_0^L f(z)g(z)dz$ with L being the length of the multi-line structure.

To complete the solution of the matrix equations thus formed, boundary conditions must be applied. Let $Z_{ref}^0=diag\left(Z_{ref}^{01},Z_{ref}^{02},\ldots Z_{ref}^{0N}\right)$ and $Z_{ref}^L=diag\left(Z_{ref}^{L1},Z_{ref}^{L2},\ldots Z_{ref}^{LN}\right)$ be the normalizing impedance matrices at $z=0$ and $z=L$, respectively. To compute the scattering parameters of the structure in this reference system, a unit excitation voltage source is placed sequentially at each port while all other ports are terminated by their proper reference impedances. For each such configuration the system of $2N*N_g$ equations is solved and the S-parameters are extracted from the voltage solution by: $S_{ij}=\dfrac{2V_{ij}-E_{ij}}{E_{ij}}$ where V_{ij} is the voltage at port i when port j is excited and E_{ij} is the excitation voltage, $E_{ij}=1$ when $i=j$, Eij=0, when $i\neq j$.

RESULTS

To illustrate the accuracy, the efficiency and robustness of the proposed approach, several non-uniform structures were analyzed. First, a linear microstrip line taper, with dimensions as shown in figure 1, has been analyzed to validate the approach. The results shown in figure 2 converge to those obtained through the MDS[5] model over the entire frequency band with as little as three propagation constants. Although this convergence behavior for single lines is similar to that of the approach used in [4], i.e., three propagation constants are also needed in [4], the resulting matrix size is divided by two in the present approach. Furthermore, the condition number of the resulting matrix is several orders of magnitude better with the present approach. This leads to a much more robust matrix solution.

However, the advantages of the present approach can be best seen by considering multiple coupled lines. By expanding the voltage only, the number of unknowns is divided by two. Since a full matrix inversion is O(N^3) operations, this will result in reduction by 8 of the number operations. Furthermore, by eliminating redundancy from the system, the condition number of the resulting matrix is improved significantly. To illustrate this, consider the two coupled microstrip lines analyzed by [3] and shown in figure 3. Figure 4 shows the results obtained using the present approach with 2, 3 or 4 basis functions. The corresponding results with both I and V expanded, as per equation (2), are shown in Figure 5. The resulting reciprocal condition number is plotted in Figure 6 for both cases. Looking at Figures 4, 5 and 6, we clearly see the superiority of the present approach in terms of its rapid convergence, stability and improved condition number in addition to the speed up resulting from using half of the number of unknowns. With such stability and efficiency the present approach can be used as a CAD tool for designing a wide range of microwave components such as wide band couplers and filtering structures. Furthermore, its accuracy over a wide range of frequencies, with relatively few basis functions, make it attractive for non-uniform interconnect analysis both in the frequency and time domains. More results for three, four and more coupled lines will be presented.

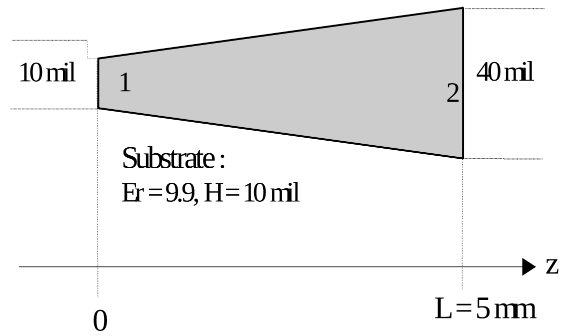
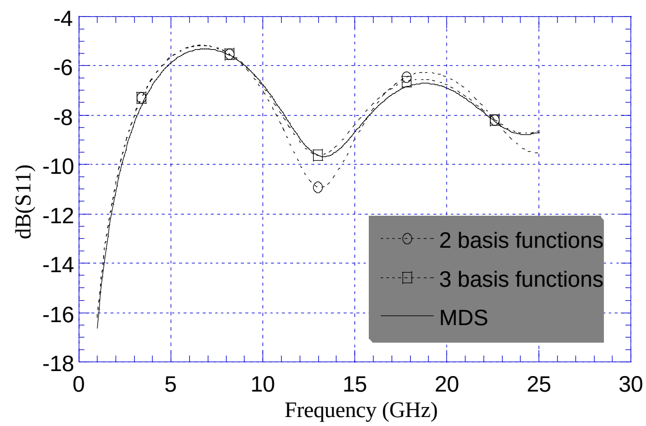
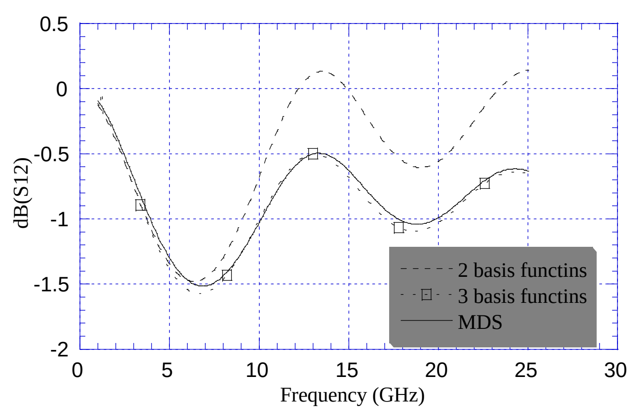


Figure 1: geometry of tapered microstrip line



(a) S11



(b) S12

Figure 2: Scattering parameters for the linear microstrip taper of figure 1.

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[3] Jen-Tsai Kuo, “Riccati Matrix Differential Equation Formulation for the Analysis of Nonuniform Multiple Microstrip Lines ” MTT Vol. 44, No. 6, pp. 880-886, June 1996.

[4] A. B. Kouki, A. Hamade and F. M. Ghannouchi, “A new formulation for the analysis of nonuniform transmission lines using frequency-varying basis functions”

[5] HP 8510B Microwave Design System (MDS), Building and Analyzing Circuits.

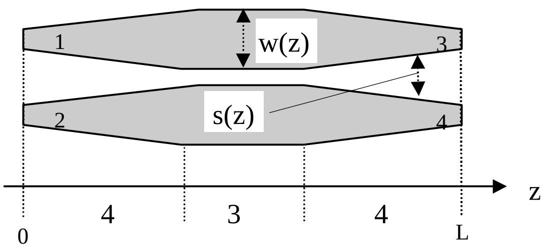
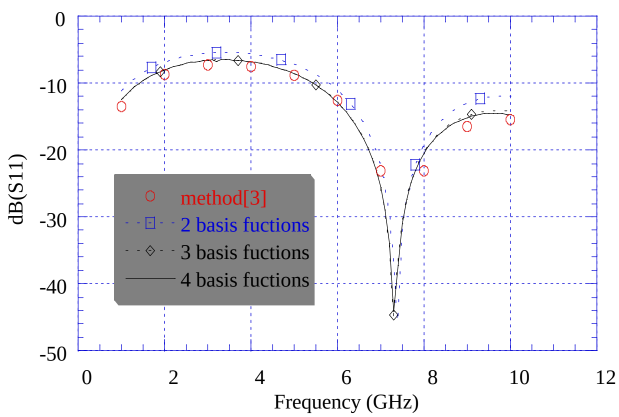
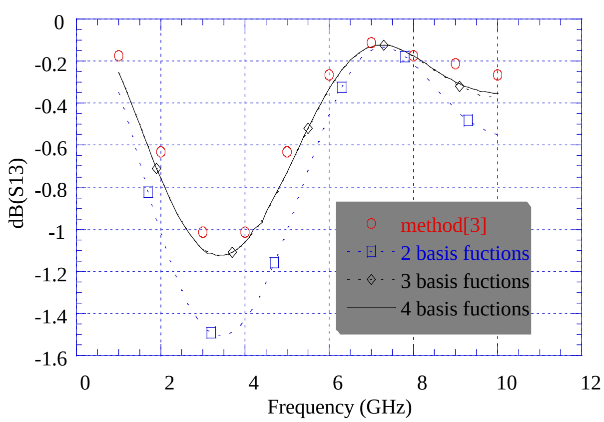


Figure 3: Geometry of the two coupled microstrip lines
Er = 9.9, H= 10 mil, L= 11 mm, W(z) = (10 + 5z) mil,
S(z) = (35 – 3.75z) mil.

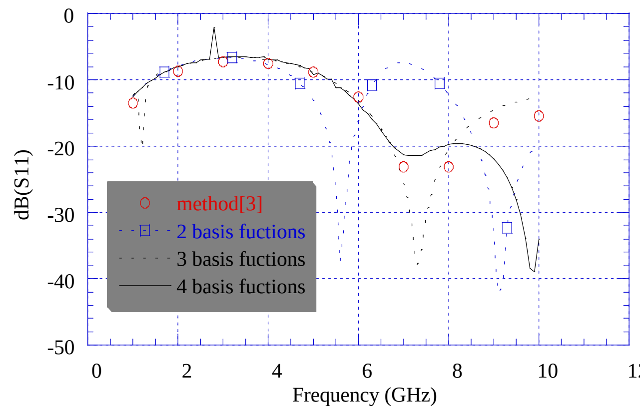


(a) S11

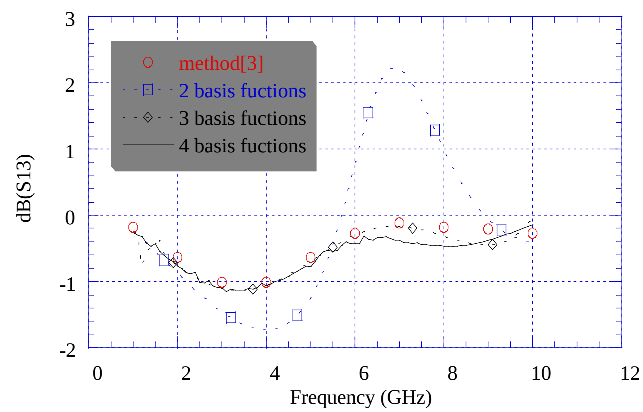


(b) S13

Figure 4: Scattering parameters of the two coupled microstrip lines with only V expended.



(a) S11



(b) S13

Figure 5: Scattering parameters of the two coupled microstrip lines with both V and I expended.

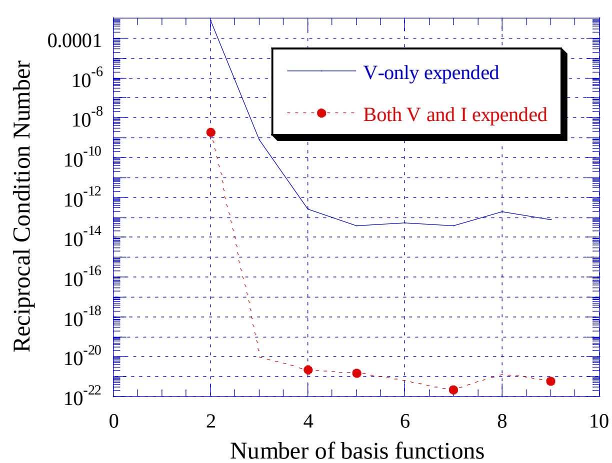


Figure 6: Reciprocal condition number of the two described approaches.